Relational Inertial Induction on Compact 3-Spheres: Deriving the Baryonic Tully–Fisher Limit from Cosmic Horizon Saturation

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Abstract

In standard physics, inertial mass is an axiomatic, unexplained internal property of isolated bodies ($F = m a$). When Keplerian mechanics fails to reproduce the flat rotation curves of galaxies and the Baryonic Tully–Fisher Relation ($v^4 = G M a_0$), mainstream astrophysics introduces massive halos of undetected Cold Dark Matter particles. In this paper, we revisit Ernst Mach's principle and Dennis Sciama's (1953) gravitoelectromagnetic (GEM) model of relational inertia, placing it for the first time within a closed, compact 3-sphere vacuum resonator $S^3(R)$ with stationary radius $R = \sqrt{6} c / H_0 \approx 3.285 \times 10^{26}\,\text{m}$.

We associate every accelerating body with its causal kinematic formation length / Rindler horizon $L_a = c^2/a$. By integrating the gravitoelectromagnetic inductive backreaction across the compact 3-sphere with volume element $\mathrm{d}V = 4\pi R^3 \sin^2\chi\,\mathrm{d}\chi$, we obtain two exact analytic regimes: 1. **The High-Acceleration (UV) Limit ($a \gg a_\star = c^2/R$):** The kinematic horizon is local ($L_a \ll R$). The induction integral evaluates to $I_{\text{UV}} = \frac{1}{2}[\gamma + \ln(2\pi) - \text{Ci}(2\pi)] \approx 1.2188267$. Under the Sciama clothes condition on cosmic mass M_U, the backreaction exactly balances the applied force: $\mathbf{F} = m_0 \mathbf{a}$, identically recovering Newtonian mechanics in the solar system, laboratory experiments, and binary pulsars. 2. **The Low-Acceleration (IR) Limit ($a \le a_\star / \pi$):** When acceleration drops below $a_\star / \pi \approx 0.87 \times 10^{-10}\,\text{m/s}^2$, the formation horizon exceeds the antipodal geodesic circumference (πR). The inductive kernel saturates uniformly across the *entire* compact 3-sphere, evaluating analytically to $J_{S^3} = \pi / (2 \chi_a)$.

We derive the **universal S^3 Mach geometric factor**: [C_{\text{mach}} = \frac{\pi}{2 I_{\text{UV}}} \approx \mathbf{1.28877742116},] and the effective relational acceleration scale: [a_{\star,\text{eff}} =

\frac{a_\star}{C_{\text{mach}}} \approx \mathbf{2.12273 \times 10^{-10}\,\text{m/s}^2} \quad \left(1.769\times a_0\right).] In this saturated regime, inertial resistance softens linearly ($m_{\text{eff}} = m_0 (a / a_{\star,\text{eff}})$), yielding the equation of motion $m_0 (a^2 / a_{\star,\text{eff}}) = G M m_0 / r^2$, which directly derives the **Baryonic Tully–Fisher Relation**: [v^4 = G M a_{\star,\text{eff}},] without fine-tuning or dark matter halos. We analyze the resulting kinematics across SPARC benchmark galaxies and delineate the strict ledger boundaries: modified inertia governs massive particles, while complementary bulk dielectric deformation is required to account for photon weak lensing (`PASS_FT_MA_LONE_KILL_PHOTON_PHI`).

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1. Introduction: Mach's Principle and the Relational Origin of Inertia

Newtonian mechanics and General Relativity postulate inertia as an intrinsic scalar constant of isolated mass: an object resists acceleration because it possesses mass m_0. In 1883, Ernst Mach challenged this premise, proposing that the inertia of any body is not an intrinsic property, but the dynamical consequence of its mutual gravitational interaction with all the other matter in the universe.

In 1953, Dennis Sciama formulated a concrete, Maxwell-like gravitoelectromagnetic (GEM) realization of Mach's principle. In Sciama's model, when a body accelerates with proper acceleration $\mathbf{a}$, the cosmic mass distribution in the body's instantaneous rest frame appears to accelerate in the opposite direction ($-\mathbf{a}$), inducing a gravitational vector potential: [\mathbf{A} = \frac{G}{c^2} \int \frac{\rho \mathbf{v}*{\text{ret}}}{r} \mathrm{d}V.] The time-derivative of this potential creates an inductive gravitoelectric field: [\mathbf{E}*{\text{ind}} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \approx \frac{G}{c^2} \left(\int \frac{\rho}{r} \mathrm{d}V \right) \mathbf{a} = \frac{\Phi_{\text{cosmic}}}{c^2} \mathbf{a}.] If the cosmic gravitational potential satisfies the cosmological "clothes" condition $\Phi_{\text{cosmic}} = c^2$, then $\mathbf{E}*{\text{ind}} = \mathbf{a}$. An applied force $\mathbf{F}$ must overcome this inductive cosmic drag: [\mathbf{F} - m_0 \mathbf{E}*{\text{ind}} = 0 \implies \mathbf{F} = m_0 \mathbf{a}.] Newton's second law is thus not an axiom, but an induction theorem.

However, Sciama assumed an infinite, flat Euclidean universe with an arbitrary spatial cutoff. In this paper, we place Sciama's induction on the mathematically rigorous foundation of the closed 3-sphere vacuum resonator $S^3(R)$ within the CPT framework.

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2. Cosmic GEM Induction on the Compact 3-Sphere $S^3(R)$

Consider a compact 3-sphere $S^3(R)$ with stationary radius $R = \sqrt{6} c / H_0 \approx 3.28524 \times 10^{26}\,\text{m}$ (§0bn1189) and uniform cosmic matter density: [\rho_0 = \frac{M_U}{2\pi^2 R^3},] where M_U is the total mass of the container.

In hyperspherical geodesic coordinates, the line element is: [\mathrm{d}s^2 = R^2 \left[\mathrm{d}\chi^2 + \sin^2\chi \left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \right) \right], \quad \chi \in [0, \pi].] The volume element is: [\mathrm{d}V = 4\pi R^3 \sin^2\chi\,\mathrm{d}\chi.]

2.1 The Kinematic Formation Floor / Rindler Horizon

Every accelerated probe possesses a causal kinematic formation length: [L_a = \frac{c^2}{a},] representing the distance to the probe's Rindler event horizon. Within the CPT framework (`PASS_FT_L3_PAIR_KILL`), field excitations with causal delay exceeding L_a cannot establish coherent retarded phase backreaction. The effective distance kernel is therefore bounded from below: [r_{\text{eff}} = \max(r, L_a) = R \max(\chi, \chi_a), \quad \text{where } \chi_a = \frac{L_a}{R} = \frac{c^2}{a R} = \frac{a_\star}{a},] and $a_\star = c^2/R \approx 2.73573 \times 10^{-10}\,\text{m/s}^2$ is the bare container acceleration.

2.2 The Relational Induction Integral

The induced gravitoelectric acceleration field is: [\mathbf{E}*{\text{ind}} = \frac{G}{c^2} I(L_a) \mathbf{a},] where the cosmic induction integral is: [I(L_a) = \int*{S^3} \frac{\rho_0}{\max(r, L_a)} \mathrm{d}V = \frac{M_U}{2\pi^2 R^3} \int_0^\pi \frac{4\pi R^3 \sin^2\chi}{R \max(\chi, \chi_a)} \mathrm{d}\chi = \frac{2 M_U}{\pi R} J_{S^3}(\chi_a),] with the dimensionless integral: [J_{S^3}(\chi_a) = \int_0^\pi \frac{\sin^2\chi}{\max(\chi, \chi_a)} \mathrm{d}\chi.]

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3. The Ultraviolet Limit: Recovery of Exact Newtonian Mechanics

When acceleration is high ($a \gg a_\star$, such as in the Solar System or terrestrial laboratories where $a \sim 10^{-3} - 10^1\,\text{m/s}^2 \gg 10^{-10}\,\text{m/s}^2$), the formation horizon is negligible on cosmic scales ($\chi_a \to 0$).

The integral reduces to: [J_{S^3}(0) = \int_0^\pi \frac{\sin^2\chi}{\chi} \mathrm{d}\chi \equiv I_{\text{UV}}.] Using the trigonometric identity $\sin^2\chi = \frac{1 - \cos(2\chi)}{2}$: [I_{\text{UV}} = \frac{1}{2} \int_0^\pi \frac{1 - \cos(2\chi)}{\chi} \mathrm{d}\chi = \frac{1}{2} \int_0^{2\pi} \frac{1 - \cos u}{u} \mathrm{d}u.] This evaluates in terms of the cosine integral function $\text{Ci}(x)$ and the Euler–Mascheroni constant $\gamma \approx 0.5772156649$: [I_{\text{UV}} = \frac{1}{2} \left[\gamma + \ln(2\pi) - \text{Ci}(2\pi) \right] \approx \mathbf{1.21882669653}.]

The Sciama Clothes Normalisation

Under the cosmological clothes condition on the container: [\frac{2 G M_U}{\pi R c^2} I_{\text{UV}} = 1 \implies M_U = \frac{\pi R c^2}{2 G I_{\text{UV}}},] we obtain identically: [\mathbf{E}*{\text{ind}} = *mathbf{a}* *implies* *mathbf{F}*{\text{applied}} = m_0 \mathbf{a}.] Newton's second law is recovered exactly without residual corrections in high-acceleration environments.

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4. The Infrared Limit: Deep-MOND Relational Softening

When acceleration drops below the container saturation threshold: [a \le \frac{a_\star}{\pi} \approx \mathbf{8.708 \times 10^{-11}\,\text{m/s}^2},] the kinematic horizon $L_a = c^2/a \ge \pi R$ exceeds the antipodal geodesic circumference of the closed 3-sphere!

Throughout the *entire* compact manifold, the kernel saturates uniformly: [\max(\chi, \chi_a) \equiv \chi_a \quad \forall \chi \in [0, \pi].] The integral evaluates in closed form: [J_{S^3}(\chi_a) = \frac{1}{\chi_a} \int_0^\pi \sin^2\chi\,\mathrm{d}\chi = \frac{1}{\chi_a} \left[\frac{\chi}{2} - \frac{\sin(2\chi)}{4} \right]_0^\pi = \frac{\pi}{2 \chi_a} = \frac{\pi R a}{2 c^2}.]

4.1 The S^3 Mach Geometric Factor

The ratio of infrared induction to the ultraviolet baseline defines the relational interpolating response: [\mu_{S^3}(a) = \frac{J_{S^3}(\chi_a)}{I_{\text{UV}}} = \left(\frac{\pi}{2 I_{\text{UV}}} \right) \frac{1}{\chi_a} = C_{\text{mach}} \left(\frac{a}{a_\star} \right),] where the **exact S^3 Mach geometric factor** is: [C_{\text{mach}} = \frac{\pi}{2 I_{\text{UV}}} = \frac{\pi}{\gamma + \ln(2\pi) - \text{Ci}(2\pi)} \approx \mathbf{1.28877742116}.]

4.2 The Effective Deep Acceleration Scale

Defining the effective acceleration scale: [a_{\star,\text{eff}} = \frac{a_\star}{C_{\text{mach}}} = \frac{c^2}{R\,C_{\text{mach}}} \approx \mathbf{2.12273 \times 10^{-10}\,\text{m/s}^2}.] The effective inertial mass softens linearly with acceleration: [m_{\text{eff}}(a) = m_0 \mu_{S^3}(a) = m_0 \left(\frac{a}{a_{\star,\text{eff}}} \right).]

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5. Derivation of the Baryonic Tully–Fisher Relation

In galactic outskirts, gravitational force remains strictly Newtonian: [F_N = \frac{G M_{\text{bar}} m_0}{r^2}.] However, the inertial resistance of the orbiting matter has softened to $m_{\text{eff}}(a) =

m_0 (a / a_{\star,\text{eff}})$. The equation of motion is: [m_0 \left(\frac{a^2}{a_{\star,\text{eff}}} \right) = \frac{G M_{\text{bar}} m_0}{r^2} \implies a = \frac{\sqrt{G M_{\text{bar}} a_{\star,\text{eff}}}}{r}.] For a circular orbit with orbital velocity v, the centripetal acceleration is $a = v^2/r$: [\frac{v^2}{r} = \frac{\sqrt{G M_{\text{bar}} a_{\star,\text{eff}}}}{r} \implies \mathbf{v^4 = G M_{\text{bar}} a_{\star,\text{eff}}}.] The flat rotation velocity $v = (G M_{\text{bar}} a_{\star,\text{eff}})^{1/4}$ is derived directly from the global geometry of $S^3(R)$ without dark matter!

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6. SPARC Benchmark Kinematics

Evaluating the deep root $v = (G M_{\text{bar}} a_{\star,\text{eff}})^{1/4}$ across four representative galaxies: * **Milky Way ($5.0 \times 10^{10} M_\odot$):** Keplerian $119.7\,\text{km/s} \to$ Derived $v = \mathbf{193.7\,\text{km/s}}$ (Obs: $220.0\,\text{km/s}$, 88%). * **NGC 3198 ($3.4 \times 10^{10} M_\odot$):** Keplerian $69.8\,\text{km/s} \to$ Derived $v = \mathbf{175.9\,\text{km/s}}$ (Obs: $150.0\,\text{km/s}$, 117%). * **NGC 2841 ($1.1 \times 10^{11} M_\odot$):** Keplerian $108.8\,\text{km/s} \to$ Derived $v = \mathbf{235.9\,\text{km/s}}$ (Obs: $285.0\,\text{km/s}$, 83%). * **DDO 154 Gas Dwarf ($3.0 \times 10^8 M_\odot$):** Keplerian $16.1\,\text{km/s} \to$ Derived $v = \mathbf{53.9\,\text{km/s}}$ (Obs: $50.0\,\text{km/s}$, 108%).

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7. Strict Ledger Integrity and Demarcation

In keeping with the CPT framework's commitment to ledger honesty: 1. **The Photon Weak-Lensing Demarcation (`PASS_FT_MA_LONE_KILL_PHOTON_PHI`):** Pure relational modified inertia operates exclusively on massive bodies ($m_0 > 0$). Because photons have $m = 0$, they do not experience inertial softening. KiDS/SDSS weak lensing proves that photons trace extra potential Φ, demanding the complementary 4D bulk dielectric medium response developed in Paper 1. 2. **Formation Floor Status (`PASS_FT_L3_PAIR_KILL`):** The kinematic horizon $L_a = c^2/a$ is an assumed/parked causal cutoff (`PASS_FT_SCIAMA_FLOOR_PARK`), not derived from 2-seat Maxwell delay. 3. **Gate 4 Non-Circularity (§0bn1189):** Laboratory G remains an external empirical input; M_U is fixed by the clothes condition, and G is not circularly returned. 4. **Governance Fences:** Live line at §0bn1190, Public Doctrine v1.1.11 unedited, Constitutive Prize remains OPEN (`PASS_FT_CONST_PRIZE_OPEN`), and Token BN1 remains unpaid **[O]**.

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